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The Explanatory Power of Phase Spaces†

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David Malament argued that Hartry Field's nominalisation program is unlikely to be able to deal with non-space-time theories such as phase-space theories. We give a specific example of such a phase-space theory and argue that this presentation of the theory delivers explanations that are not available in the classical presentation of the theory. This suggests that even if phase-space theories can be nominalised, the resulting theory will not have the explanatory power of the original. Phase-space theories thus raise problems for nominalists that go beyond Malament's initial concerns.

1. Introduction

In one of the first discussions of Hartry Field's *Science Without Numbers*, David Malament raised a number of objections that set the agenda for much of the subsequent debate. In our view, one of these objections has been underappreciated. The objection in question concerns the plausibility of nominalising non-space-time theories such as phase-space theories. A great deal of attention has been focused on the question of whether quantum mechanics, with its underlying Hilbert spaces, can be nominalised. But few seem concerned about the status of Hamiltonian formulations of classical theories and their reliance on phase spaces. The problem, in a nutshell, is that phase spaces are spaces of possible, but mostly non-actual, initial conditions. These are not the kind of entities that even liberal nominalists like Field are able to accept. So first we explore the use of phase spaces in physics, and we do this by way of an example: the Hénon-Heiles system. We show that although there is a Lagrangian formulation of the theory in question that does not employ phase spaces, the cost of adopting such an approach is a loss of explanatory power.

Our example also sheds light on another debate in the recent philosophy-of-mathematics literature: the issue of whether mathematics

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can provide explanations of physical phenomena. One of the present authors has argued elsewhere that mathematics can unify physical theories [Colyvan, 2001; 2002] and in some cases mathematics can offer genuine explanations [Colyvan, 2001; forthcoming]. (See also [Baker, 2005], [Melia, 2002], and [Steiner, 1978a] on this issue.) The reason this is important is that if mathematics can offer explanations of physical phenomena, the nominalist's task is all the more difficult. After all, for scientific realists such as Hartry Field, who accept inference to the best explanation, dispensing with entities that are genuinely explanatory is no easy matter. We will argue that our example suggests that Hamiltonian formulations of theories (with their underlying phase spaces) are able to explain certain features of the physical system in question that the alternative (Lagrangian) formulation does not seem able to explain. A nominalist alternative to a physical theory that lacks the explanatory power of its Platonist counterpart is no alternative at all. Putting these two points together suggests that Hamiltonian formulations of classical theories present genuine problems for at least Field-style nominalisation programs.

Before we start on Malament's objection and our main example, let us prepare the ground a little by presenting an example of how mathematics might be thought to be offering explanations of physical (in this case biological) phenomena. Start with the question of why hive-bee honeycomb has a hexagonal structure. What needs explaining here is why the honeycomb is always divided up into hexagons and not some other polygon (such as triangles or squares), or any combination of different (concave or convex) polygons.[1] Biologists assume that hive-bees minimise the amount of wax they use to build their combs, since there is an evolutionary advantage in doing so. For example, in the *Origin of Species* Darwin writes:

> [...] that individual swarm which thus made the best cells with least labour, and least waste of honey in the secretion of wax, having succeeded best, and having transmitted their newly-acquired economical instincts to new swarms, which in their turn will have had the best chance of succeeding in the struggle for existence. [Darwin, 1998, p. 350]

So the biological part of the explanation is that those bees which minimise the amount of wax they use to build their combs tend to be selected over bees that waste energy by building combs with excessive amounts of wax. The mathematical part of the explanation then comes from what is known as the honeycomb conjecture: *a hexagonal grid represents the*

[1] A convex polygon is one which contains any line segment between its points, and a concave polygon is any polygon which does not satisfy this condition.

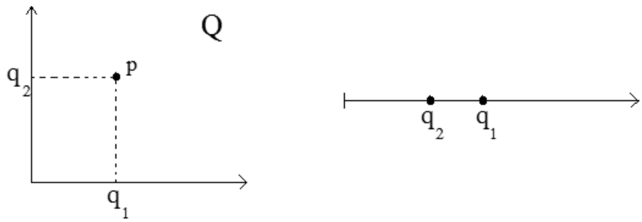

FIG. 1. The configuration space (left) for a two-particle system (right).

best way to divide a surface into regions of equal area with the least total perimeter.[2] This conjecture was proved in 1999 by Thomas C. Hales [2001], and this proof explains why a hexagonal grid is the optimal way to divide a surface up into regions of equal area.[3] So the honeycomb conjecture (now the honeycomb theorem), coupled with the evolutionary part of the explanation, explains why the hive-bee divides the honeycomb up into hexagons rather than some other shape, and it is arguably our best explanation for this phenomenon.

The explanatory role that mathematics plays in science will be of crucial importance to the purposes of this paper, and so it will be useful to keep it in mind until we return to the issue again at the end of section two. The point is that Field accepts the principle of inference to the best explanation [1989, pp. 15–16]; so any nominalist reformulation of our scientific theories that he provides must have at least the same explanatory resources as the Platonist counterparts. In section three we will argue that even if Field is successful in providing a nominalist reformulation of our scientific theories, there are examples of mathematical explanations to which there is no non-mathematical counterpart.

[2] The honeycomb conjecture should not be confused with the circle-packing problem. The circle-packing problem concerns which arrangement of *circles* in the two-dimensional plane will constitute the densest possible packing. Gauss proved that the hexagonal arrangement was the densest packing of *circles* out of all possible regular arrangements, and László Fejes Tóth proved the more general result: that this is true for regular and irregular arrangements. The honeycomb conjecture states that the optimal way to divide the two-dimensional plane into regions (of *any* shape) of equal area with least total perimeter is the *hexagonal grid*.

[3] There are some interesting questions here about the explanatory power of proofs. Conventional wisdom has it that not all proofs are explanatory; some do and some do not *explain* the theorem in question. The idea is that proof and intra-mathematical explanation can come apart. Interesting as these issues are, we will not pause over them here. Our interest is primarily with mathematical explanation of physical phenomena (what we might think of as *extra-mathematical explanation*). See [Steiner, 1978b] for more on intra-mathematical explanations.

2. Malament's Objection

Let us begin with a review of some basic mechanics. A configuration space, Q, of a physical system, is a space whose points represent different possible configurations of that physical system. For example, the configuration space for a system containing two particles, each with one degree of freedom is $\mathbb{R}^2$. Any given state of such a system can be represented by a point in the configuration space shown in Figure 1. The point p in the configuration space Q represents the positions of particles 1 and 2 on the line relative to some frame of reference. For a system of N particles each with three degrees of freedom—that is N particles in physical space—the configuration space is

$$Q = \mathbb{R}^{3N}$$

since it takes three numbers to represent each particle's position in physical space.

The Lagrangian function L is introduced as the kinetic energy T of the system less the potential energy V:

$$L = T - V.$$

For any conservative system it can be shown from a variational principle,

$$\Delta \int L = 0$$

that:[4]

$$\frac{d}{dt}\left(\frac{\partial L}{\partial q_i}\right) - \frac{\partial L}{\partial q_i} = 0,$$

where $i = 1, 2, ..., 3N$; the q_i are $3N$ position co-ordinates, and t is time. This is the Lagrangian formulation of mechanics. For a system of N particles in physical space, the above is a system of $3N$ second-order differential equations. The Hamiltonian formulation of mechanics is introduced because it provides a more powerful and versatile method than the Lagrangian one. The Hamiltonian formulation is based on Hamilton's

[4] Conservative in the sense that the work done by a force moving an object from point A to point B is independent of the path the object takes as it moves from A to B. The gravitational and electric fields are conservative systems, whereas any system with friction or air resistance is a non-conservative system.

canonical equations of motion:

$$\dot{q}_i = \frac{dq_i}{dt} = \frac{\partial H}{\partial p_i},$$
$$\dot{p}_i = \frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i},$$
$$\frac{\partial L}{\partial t} = -\frac{\partial H}{\partial t},$$

where again $i = 1, 2, ..., 3N$; H is defined in terms of the Lagrangian L using a Legendre transformation:

$$H(q, p, t) = \sum_{i=1}^{3N} p_i \dot{q}_i - L(q, \dot{q}, t)$$

and the p_i are $3N$ generalised momentum co-ordinates. The generalised momentum co-ordinates can be obtained from the position co-ordinates and the Lagrangian using the following equation:

$$p_i = \frac{\partial L}{\partial \dot{q}_i}.$$

The space of the q and p co-ordinates specifying a physical system is known as the *phase space*. The configuration space can be thought of as the half of the phase space that contains the position co-ordinates q. In the Hamiltonian formulation of mechanics, the dynamics of a system are defined by the evolution of points, 'trajectories', in this phase space–which has $6N$ dimensions. These trajectories are defined by a vector field $v(q, p)$ in the phase space which can be formulated in terms of the Hamiltonian, H:

$$v(q, p) = \sum_{i=1}^{3N} \frac{\partial H}{\partial p_i} \cdot e_i - \sum_{i=3N+1}^{6N} \frac{\partial H}{\partial q_{i-3N}} \cdot e_i,$$

where the e_i are basis vectors for the phase space. This provides a powerful method for analysing the dynamics of physical systems. For a system of only one degree of freedom, that is, a single particle that can only move along one direction, such a vector field may look something like Figure 2. Each point in the phase space is a *possible dynamical state* of the system and has a corresponding vector, which determines how the system will evolve from that state. A possible evolution of our simple hypothetical system is shown by the curve in Figure 2.

Now we are ready for Malament's criticism of Field's project. Essentially, Malament's concern is that Field cannot nominalise any theory that makes use of such phase spaces. [Malament, 1980, p. 553]

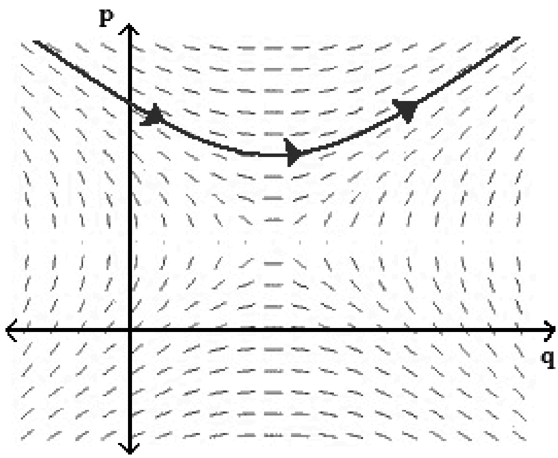

FIG. 2. A Hamiltonian vector field.

The problem is that not only are the points in phase space abstract entities, but they also *represent abstract entities*, *viz.* possible dynamical states. With Field's primary example, Newtonian gravitation theory, the abstract entities that the theory quantifies over represent physical entities, *viz.* space-time points—something Field (who is a substantivalist) has no qualms about—and so there is no problem. But there is a problem with phase spaces, Malament writes:

> [...] Even a generous nominalist like Field cannot feel entitled to quantify over possible dynamical states. The point is very simple. Suppose Field wants to give some physical theory a nominalist reformulation. Further suppose the theory determines a class of mathematical models, each of which consists of a set of 'points' together with certain mathematical structures defined on them. Field's nominalization strategy cannot be successful unless the objects represented by the points are appropriately physical (or non-abstract). In the case of classical field theories the represented objects are space-time points or regions. So Field can argue, there is no problem. But in lots of cases the represented objects are abstract. In particular this is true in all 'phase space' theories. [Malament, 1980, p. 533]

Field has shown how we can nominalise Newtonian space-time geometry and speak nominalistically about scalar and vector fields over such a geometry, but he has not shown how we can talk nominalistically about phase spaces. And as Malament points out, it does not seem that a nominalist account of phase spaces is even possible. Given that Field has shown how a nominalist account of the structure of flat space-time is possible, it seems reasonable to assume that the same *can* in fact be done for other space-times—such as ones with curvature—even though it is not clear *how* this can be done. But phase spaces are a completely different matter. They are clearly not like the manifolds we use to represent space-times; points in phase spaces are abstract objects that represent other abstract objects (possible dynamical states), whereas the points in manifolds are abstract objects that represent other concrete objects (points in space-time). So applying the kind of representation theorems that Field employs to deal with Newtonian space-time clearly will not do the trick.

The nominalist might claim, in response to Malament's criticism, that Hamiltonian formulations in phase spaces do not add any new physics; they are just simply another way of expressing the same thing that Field has shown can be nominalised. This is certainly true for classical Hamiltonian mechanics, which was the object of Malament's first objection. But, as Malament goes on to stress, when we move to theories such as quantum mechanics, where we use Hilbert spaces, this reply no longer seems reasonable. There has been some work on giving nominalist accounts for the structure of Hilbert spaces, but it is not clear that this has been formally successful, and the works we have in mind quantify over such objects as 'physically real propensities'—which are nominalistically suspicious to say the least. In any case, we will not focus on such issues in this paper.[5] We want to press the original Malament objection a bit further. Let us grant that the nominalist response to Malament is right; that is, we assume that phase-space theories do not add any new physics to the picture, and so any physical law that can be stated in terms of points in phase space has an equivalent nominalistic counterpart. Even granting this, there is another important role that phase-space theories play in science apart from their ability to provide a neat expression of the relevant laws of physics; they play an explanatory role.

In the next section we will argue that phase spaces have explanatory resources that their nominalised counterparts (whatever they may be) do not share. This fact, coupled with the principle of inference to best explanation, will result in an ontological commitment to mathematical entities despite the (assumed) success of a Field-style nominalisation of phase spaces.[6]

[5] See [Balaguer, 1996] and [Bueno, 2003] for discussion on nominalising mechanics.

[6] Though even without invoking inference to the best explanation, there are problems for the nominalist, since the Platonist theory has greater explanatory power and thus would seem to be the better theory.

3. Explanatory Power

Our argument will be by way of an example. We will look at an analysis of a physical system using phase spaces that will give us an understanding of that system which cannot obviously be obtained otherwise. The physical system we will look at is known as the Hénon-Heiles system. It was introduced in 1964 by Hénon and Heiles to describe the motion of a star around a galactic centre, where that motion is restricted to the plane.[7] Put simply, the Hénon-Heiles system is a point particle moving in the potential:

$$U(q_x, q_y) = \frac{1}{2}(q_x^2 + q_y^2) + q_y q_x^2 - \frac{1}{3}q_y^3.$$

The Hamiltonian for the Hénon-Heiles system is then:

$$\begin{aligned} H &= T + U \\ &= \frac{1}{2}(p_x^2 + p_y^2) + \frac{1}{2}(q_x^2 + q_y^2) + q_y q_x^2 - \frac{1}{3}q_y^3, \end{aligned}$$

where we have set the conjugate momentum coordinates to: $p_i = \frac{dq_i}{dt}$. The equations of motion expressed in terms of the Hamiltonian are:

$$\begin{aligned} \frac{dq_i}{dt} &= \frac{\partial H}{\partial p_i}, \\ \frac{dp_i}{dt} &= -\frac{\partial H}{\partial q_i}. \end{aligned}$$

And from these we can obtain the equations of motion in terms of the position coordinates q_x and q_y:

$$\begin{aligned} \frac{d^2 q_y}{dt^2} &= \frac{dp_y}{dt} = -\frac{\partial H}{\partial q_y} = -q_y - q_x^2 + q_y^2, \\ \frac{d^2 q_x}{dt^2} &= \frac{dp_x}{dt} = -\frac{\partial H}{\partial q_x} = -q_x - 2q_x q_y. \end{aligned}$$

Now we have two options. We can either look at the evolution of the position coordinates in the q_x-q_y plane, or we can look at the evolution of the dynamic states in the phase space. A slight complication arises for this second option because the phase space is actually 4-dimensional, *i.e.*, points in the phase space, are represented by quadruples of the form: (q_x, q_y, p_x, p_y). So if we are to analyse the system using the phase space

[7] For a more detailed analysis, see [Lichtenberg and Lieberman, 1992, pp. 46–50], or [Hénon and Heiles, 1964].

we need to look at a plane that cuts through the phase space and study the motion via the induced Poincaré map.

The Poincaré map for the system comes about when we take a plane that cuts through the space of solutions and look at how those solutions pass through the plane. When a solution passes through the plane we can think of it as making a 'dot' on the plane. The solution then 'flies' around until it passes through the plane again, where it makes another 'dot'—and it continues to do this forever, passing through the plane over and over again, each time making another 'dot'. The Poincaré map is the function that maps the first 'dot' on to the second 'dot', on to the third 'dot', and so on. Depending on the nature of the system, any given solution may pass through the plane in many different places causing many different 'dots'. All solutions will pass through the plane an infinite number of times, but some solutions will only pass through the plane in a few different spots—such solutions are periodic. Solutions that never pass through the same arbitrarily small neighborhood of a point twice are chaotic.[8]

An illustration of how the Poincaré map works for the Hénon-Heiles system is in Figures 3 and 4. Firstly, we fix the total energy of the system to be a constant, E, so that our problem is reduced to three degrees of freedom:

$$E = \frac{1}{2}(p_x^2 + p_y^2) + \frac{1}{2}(q_x^2 + q_y^2) + q_y q_x^2 - \frac{1}{3}q_y^3.$$

E is then a parameter of the system which we can vary. We then look at a particular plane in the phase space and investigate how the solutions pass through it. Figure 3 shows a particular solution of the system in the q_y-q_x plane; this corresponds to the path the star takes as it moves about the galactic centre. Figure 4 shows the same solution in the q_y-p_y-q_x volume as it passes through the $(q_x = 0)$ plane.

To state this a little more formally, we take a 2-dimensional cross section of the phase space: Γ, and define the Poincaré map: $\rho : \Gamma \to \Gamma$ such that $\rho(x) = \Phi^\tau(x)$, where Φ is the trajectory and $\tau(x) = \min\{t \in \mathbb{R} : t > 0, \Phi^\tau(x) \in \Gamma\}$ is the first time point when the trajectory passes through Γ again.

By investigating how the solutions pass through the plane, we can obtain a great deal of information about the physical system in question; information which is not obviously obtainable otherwise. This is the important point for the purposes of this paper because it suggests that the phase-space analysis has explanatory resources that the Lagrangian

[8] There are more complicated ways of classifying the types of possible solutions, but this will suffice for the purposes of this paper. See [Lichtenberg and Lieberman, 1992, pp. 42–46] for more on the different kinds of possible solutions.

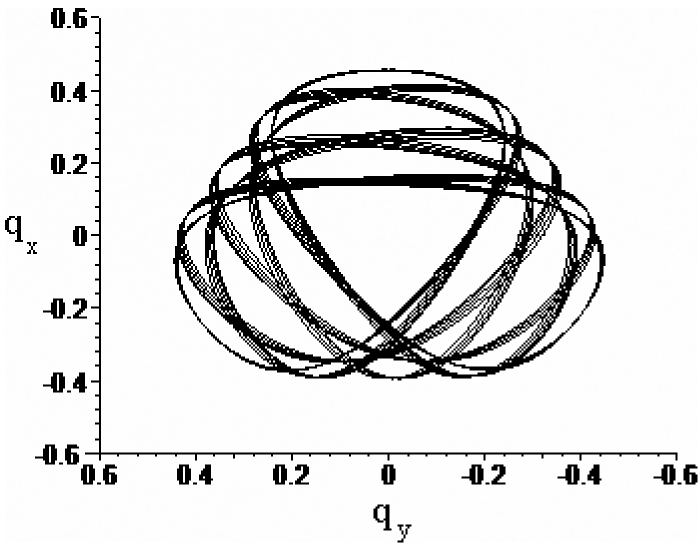

FIG. 3. A solution to the Hénon-Heiles system in the q_y-q_x plane.

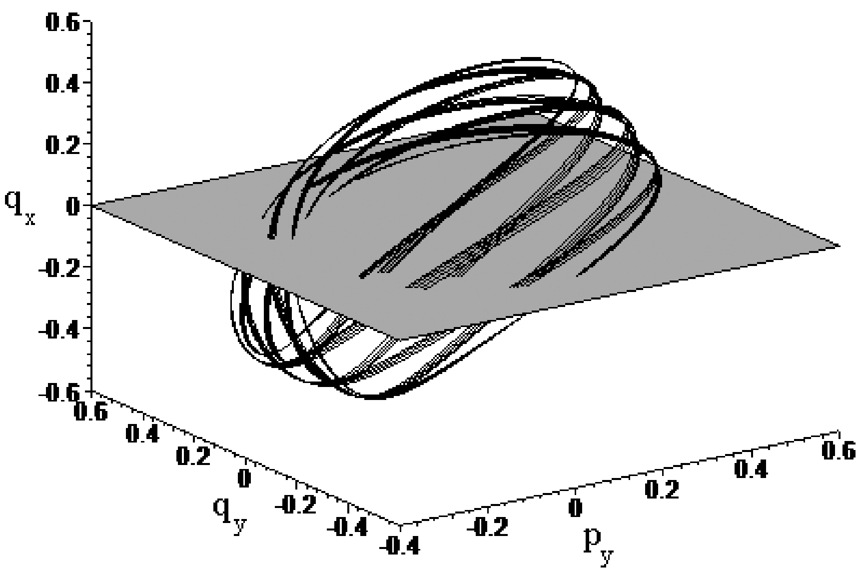

FIG. 4. A solution to the Hénon-Heiles system in the q_y-p_y-q_x volume.

analysis lacks. If this is correct, then there is a problem for Field, since science-with-numbers, it would seem, is more explanatory than science-without-numbers.

The phase space can give us a lot of information about the Hénon-Heiles system when we consider sets of possible dynamical states of the system for given particular energy levels. The *explanandum* that we will use the phase-space analysis to explain is that high-energy Hénon-Heiles systems tend to exhibit chaotic and unpredictable motion, and low-energy Hénon-Heiles systems tend to exhibit regular and predictable motion. Using the Poincaré map we can find, given a particular energy level of the system, what circumstances will give rise to regular motion and which ones give rise to chaotic motion. For example, if the total energy is set to $E = 1/8$ we get the Poincaré map on the ($q_x = 0$) plane pictured in Figure 5. This tells us which states of the system give rise to regular motion and which ones give rise to chaotic motion. A dynamic state gives rise to chaotic motion if a nearby state is mapped arbitrarily far away at some later time. So for example, if the system is ever anywhere near the dynamic state $(q_x = 0, q_y = 0.21, p_x = 0.32, p_y = -0.33)$, we know that the motion will be chaotic and unpredictable. Figure 5 illustrates how two points near this dynamic state get mapped away from each other. A dynamic state that gives rise to regular motion will have the property such that nearby dynamic states will stay close to it as they get mapped around the plane.

Something else we can discover about the system by studying the phase space is how the regions of possible dynamical states that give rise to chaotic motion change as the total energy of the system changes. For example, for the Hénon-Heiles system, we know from the Poincaré map that for high energies chaotic motion prevails, and as we decrease the total energy of the system, the regions of chaos shrink until there is very little chaotic motion left and regular motion prevails (see Figure 6 for an illustration).[9]

From this we can state some general results:

- *All galactic systems that can be modeled by the Hénon-Heiles system with low energies tend to exhibit regular and predictable motion;*
- *All galactic systems that can be modeled by the Hénon-Heiles system with high energies tend to exhibit chaotic and unpredictable motion.*

[9] This can be shown more formally using Liapounov exponents but such an analysis would take us too far afield. A computer-generated pictorial analysis will suffice for our purposes.

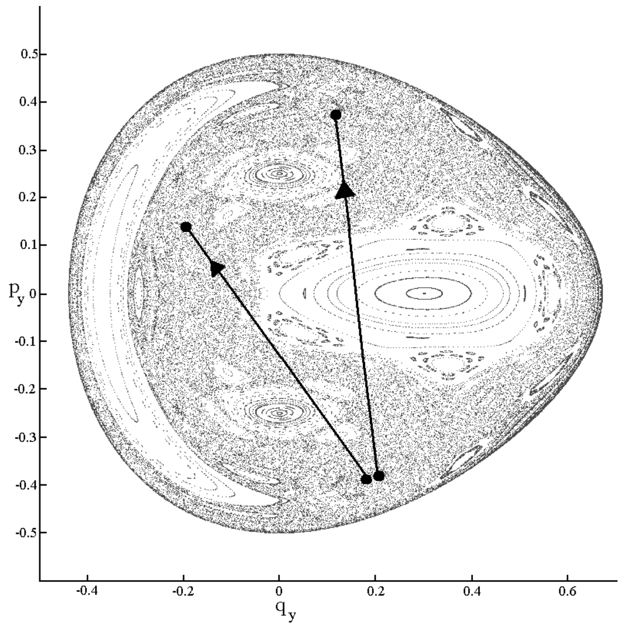

FIG. 5. Two nearby chaotic dynamic states are mapped away from each other.

So the theory of phase spaces, Poincaré maps, and differential equations explain why high-energy Hénon-Heiles systems exhibit chaotic and predictable motion and why low-energy ones exhibit regular and predictable motion. These are explanations that can be obtained from an analysis of the system using its phase space, but which cannot be obtained otherwise—at least it is not obvious how such explanations could be obtained otherwise.

We mentioned earlier that we had two options for our analysis. Instead of studying the phase space, we could look at the various paths in the q_y-q_x plane that the star can take for different energy levels. But such an analysis would be extremely tedious (to say the least) and this just does not give us the same kind of understanding as the phase-space analysis does. The explanatory power is in the structure of the phase space and the Poincaré map. So it seems that this is a case where using the phase space is essential to our understanding and ability to explain certain features of the world. Non-linear systems like the Hénon-Heiles system are abundant in the world, and our only tools for analysis of these systems are our theories of phase spaces and Poincaré maps. So even though the nominalist can translate all talk about phase spaces into

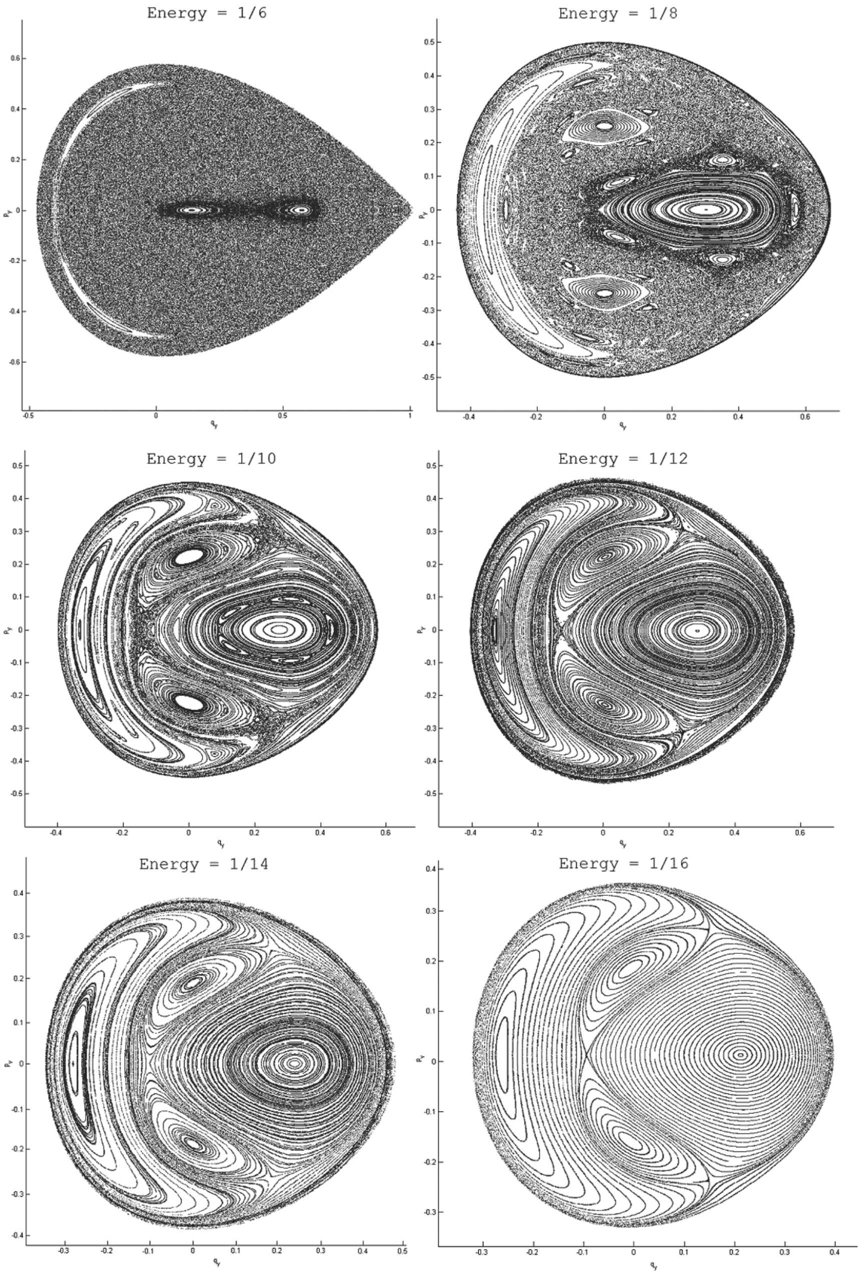

FIG. 6. For high energy ($E = 1/6$), nearly all of the dynamic states are chaotic. As the energy decreases ($E = 1/8 \rightarrow E = 1/14$) the regions of chaotic dynamic states shrink, until nearly all dynamic states are regular and predictable ($E = 1/16$).

talk about space-time points, there is loss of explanatory power in the process.[10]

4. Conclusion

It is by no means uncontroversial that mathematics can play an explanatory role in science, and even if there are good candidates for mathematical explanation (where that explanation is the *best* one for the phenomena to be explained), they do not immediately force one to be ontologically committed to mathematical entities. For an example of mathematical explanation to be of the ontologically committing type, there must be no matching nominalist explanation. For example, as we saw previously, the mathematical part of the explanation for the hexagonal structure of the hive-bee honeycomb comes from the proof of the honeycomb conjecture—a result in geometry and topology. But Field has shown how we can speak nominalistically about the geometry of (Newtonian) space-time, and so it seems likely that a similar result could be proven in his system. If this is possible, then any explanation involving the nominalist form of the honeycomb conjecture would arguably be *at least as good* as the original form of explanation presented earlier. In fact, it might be argued that such a nominalistic explanation is better than the Platonist one, since the former, but not the latter, would be intrinsic. That is, with the nominalist explanation one is 'able to explain the behaviour of the physical system *in terms of the intrinsic features of that system*, without invoking extrinsic entities (whether non-mathematical or mathematical) whose properties are irrelevant to the behaviour of the system being explained'. ([Field, 1989, p. 193], emphasis original).

The example of the Hénon-Heiles system, however, is different. The force behind this example is that, as Malament points out, it seems highly unlikely that Field can provide a nominalist account for the structure of phase spaces. Or at least nothing in Field's treatment of space-time indicates how phase spaces would be nominalised. Since he cannot do this, he does not have recourse to the tools of analysis built upon phase spaces such as the Poincaré map. Our example also illustrates that we do not have to move to branches of science such as quantum mechanics to raise difficulties for Field; classical mechanics is difficult enough. If there is a way that Field can find a matching nominalistic explanation, it might only be through an analysis of each solution in the q_y-q_x plane, but such an explanation would be, at best, extremely piecemeal. For such an

[10] The loss of explanatory power in alternative mathematical formulations of a theory may be more widespread. Mark Steiner [1978c] argues that the explanatory power of the concept of a complex number (understood as a vector in $\mathbb{R}^2$) is not preserved under the ordered-pair formulation of complex numbers.

explanation to work, large classes of solutions would have to be analysed individually in the q_y-q_x plane for a whole range of different energy levels, and even then it is not clear that this would help. To explain why unpredictable motion is more likely for higher-energy levels, the nominalist must either determine whether or not each solution is chaotic (which may still implicitly require using properties of the phase space) or show that all nearby solutions diverge from it. So it would seem that all the solutions analysed would have to be compared pairwise, making the explanation so piecemeal that one wonders whether it would still count as an explanation.

Another possible move that a Field-style nominalist might make is simply to restrict the class of explanations that are ontologically relevant to causal ones—that is, to make the principle of inference to the best explanation only apply to the best causal explanation. Since mathematical entities are acausal, mathematical explanations, even if there are genuine examples of them, do not have an impact on one's ontology. Indeed this is something Field briefly entertains, though he notes that such a position is a delicate one to maintain [Field, 1989, pp. 19–20]. It would also seem to be question-begging in the context of this debate. Indeed, a great deal of the appeal of Field's nominalist program is that he tackles the issues in question head on and does not beg important questions such as this. A restriction of inference to the best explanation to exclude non-causal explanations would undermine much of the appeal of Field's project, not to mention render a great deal of it obsolete.[11]

Our response to this is to agree that in order for the mathematical explanation to be an explanation of empirical facts, some appropriate bridge principles are required. But this does not mean that the mathematical explanation is restricted to pure mathematics. Yes, there is a great deal of work being done by the bridge principles in order for the mathematical explanations to be explanations of physical facts, and there is a great deal to be said about the nature and adequacy of these bridge principles, but this does not reduce the importance of the mathematical explanation in question. Indeed, the bridge principles in question are mappings between physical systems and mathematical structures, and so are themselves mathematical entities (*i.e.*, mappings). If the nominalist hopes to defuse the situation by having the bridge principles shoulder some of the explanatory load, this seems a poor way to proceed. First, these bridge principles do not seem to do anything more than allow the transmission of the mathematical explanations to the empirical domain. And, secondly, as we have already pointed out, the bridge principles are

[11] If a nominalist restricted inference to the best explanation to the narrower inference to the best causal explanation, much of the hard work of nominalising physical theories would be unnecessary: acausal mathematical entities would be ruled out from the start.

themselves pieces of mathematics. We can put the point this way: we have provided a mathematical explanation of the behaviour of certain physical systems, and although this explanation requires appropriate bridge principles, no alternative non-mathematical explanation is on offer. The latter is what the nominalist requires. Pointing out that the mathematics only explains the behaviour of the physical system once appropriate bridge principles have been specified is true but irrelevant.

To sum up. Given that no satisfactory nominalisation of phase spaces is forthcoming, and given that such theories are employed extensively in many branches of modern science, there would appear to be a problem here for a Field-style nominalisation program. That was Malament's largely overlooked objection to Field's program. This objection strikes us as a good one, despite the possibility of nominalising the theories in question via their Lagrangian formulations. We have pushed the point further, arguing that even if nominalisation via this route is possible, the resulting theory is likely to be less explanatory; there is explanatory power in phase-space formulations of theories, and this explanatory power does not seem recoverable in alternative formulations.

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